Revision 6.0

There are three cases (and formulas). First the attacker kills defender, second the defender kills the attacker, third the attacker withraws from combat before dying.

$$P(U = n_A, V = N_D) \times (t + w + 1) = P([X = H_A - n_A d_D > 0]\&[Y = H_D - N_D d_A \leq 0]) \times (t + w + 1)$$

$$= \sum_{i=\max\{1,-s-t\}}^{w-s} \sum_{j=0}^{i} \left(n_A \geq j \ ? \ \binom{i}{j} P_D^j P_A^{i-j} \frac{(N_D + n_A - j - 1)!}{(n_A - j)!(N_D - 1)!} P_D^{(n_A - j)} P_A^{N_D} \ : \ 0 \right)$$

$$+ \sum_{i=\max\{0,s-w\}}^{s+t} \sum_{j=0}^{i} \binom{i}{j} P_A^j P_D^{i-j} \left(N_D > j \ ? \ \frac{(n_A + N_D - j - 1)!}{(N_D - j - 1)!(n_A)!} P_A^{(N_D - j)} P_D^{n_A} \ : \ (n_A = 0 \ ? \ 1 \ : \ 0) \right).$$

$$P(U = N_A, V = n_D) \times (t + w + 1) = P([X = H_A - N_A d_D \leq 0]\&[Y = H_D - n_D d_A > 0]) \times (t + w + 1)$$

$$= (1-Q) \sum_{i=\max\{1,-s-t\}}^{w-s} \sum_{j=0}^{i} \binom{i}{j} P_D^j P_A^{i-j} \left(N_A > j \ ? \ \frac{(n_D + N_A - j - 1)!}{(N_A - j - 1)!(n_D)!} P_D^{(N_A - j)} P_A^{n_D} \ : \ (n_D = 0 \ ? \ 1 \ : \ 0) \right)$$

$$+(1-Q) \sum_{i=\max\{0,s-w\}}^{s+t} \sum_{j=0}^{i} \left(n_D \geq j \ ? \ \binom{i}{j} P_A^j P_D^{i-j} \frac{(N_A + n_D - j - 1)!}{(n_D - j)!(N_A - 1)!} P_A^{(n_D - j)} P_D^{N_A} \ : \ 0 \right).$$

$$P(U = N_A - 1, V = n_D) \times (t+w+1) = P([X = H_A - (N_A - 1) d_D \leq 0]\&[Y = H_D - n_D d_A > 0]) \times (t+w+1)$$

$$= Q \sum_{i=\max\{1,-s-t\}}^{w-s} \sum_{j=0}^{i} \binom{i}{j} P_D^j P_A^{i-j} \left(N_A > j \ ? \ \frac{(n_D + N_A - j - 1)!}{(N_A - j - 1)!(n_D)!} P_D^{(N_A - j)} P_A^{n_D} \ : \ (n_D = 0 \ ? \ 1 \ : \ 0) \right)$$

$$+Q \sum_{i=\max\{0,s-w\}}^{s+t} \sum_{j=0}^{i} \left(n_D \geq j \ ? \ \binom{i}{j} P_A^j P_D^{i-j} \frac{(N_A + n_D - j - 1)!}{(n_D - j)!(N_A - 1)!} P_A^{(n_D - j)} P_D^{N_A} \ : \ 0 \right).$$

where $P()$ denotes the probability of an event,
P_A (P_D) is the probability of the attacker (defender) winning a round of combat,
s is the number of attacker's first strikes minus the defender's first strikes (can be negative),
t (w) is the number of first strike *chances* for the attacker (defender),
H_A (H_D) is the number of hit points the attacker (defender) has at the start of combat,
d_A (d_D) is the amount of damage (in hit points) dealt by the attacker (defender) when he wins a round of combat,
$N_A = \lceil \frac{H_A}{d_D} \rceil$ is the number of hits it would take to kill the attacker (similarly $N_D = \lceil \frac{H_D}{d_A} \rceil$ is the number of hits it would take to kill the defender),
U (V) is the random variable representing the number of hits taken by the attacker (defender),
X (Y) is the random variable representing the hit points the attacker (defender) has remaining at the end of combat,
$(N_D > i)$ is a boolean expression evaluated to 1 if true and 0 if false (when false, the summation over i can be discontinued),
$n_A \in \{0, 1, \ldots, N_A - 1\}$, $n_D \in \{0, 1, \ldots, N_D - 1\}$,
Q is the retreat odds for the *attacker* if applicable.

Note that all of d_A, d_D, P_A and P_D can be calculated as in Arathorn's article. That is, if A is the modified attacker's strength and D the modified defender's strength, so $R = \frac{A}{D}$ is the all important *strength ratio*, then

$$P_A = \frac{R}{1 + R}$$

$$P_D = \frac{1}{1+R}$$

$$d_A = \lfloor \frac{20(3R+1)}{3+R} \rfloor$$

$$d_D = \lfloor \frac{20(3+R)}{3R+1} \rfloor$$

The red formulas are outdated.

n_A and n_D are variables that effectively represent the number of combat rounds the victor actually loses. For example, if we take the first formula and plug in $n_A = 0$ we would calculate the probability that the attacker wins the combat AND sustains no damage whatsoever. As you'd expect for most battles this will usually be a very small number. The conditions on n_A and n_D are simply there to ensure the victor does not also die in battle - it is impossible for both combatants to die!

Note we could also define for clarity $N = N_A + N_D$ as the number of possible outcomes of battle (we ignore the specific order in which damage was dealt - just the end result is what we want). For example, two fully healthy (100HP) units with equal modified strengths would give $R = 1$. This means they would deal 20 hit points damage in each round of combat. It should (hopefully) be easy to see there are 10 possible outcomes (100,80,60,40,20 hit points for the victor and one of two possible victors).

Therefore if we wanted to calculate the probability that the attacker wins the combat, we would take the first formula and plug in each of $n_A = 0, 1, 2, 3, 4$ and then take the sum of those 5 results.

When there are no first strikes or retreat chances involved, the formulas simplify to

$$\begin{aligned} P(U = n_A, V = N_D) &= P_{s,t}([X = H_A - n_A d_D > 0] \& [Y = H_D - N_D d_A \leq 0]) \\ &= \frac{(n_A + N_D - 1)!}{(N_D - 1)! n_A!} (P_A)^{N_D} (P_D)^{n_A} \end{aligned}$$

$$\begin{aligned} P(U = N_A, V = n_D) &= P_{s,t}([X = H_A - N_A d_D \leq 0] \& [Y = H_D - n_D d_A > 0]) \\ &= \frac{(N_A + n_D - 1)!}{n_D! (N_A - 1)!} (P_A)^{n_D} (P_D)^{N_A} \end{aligned}$$

The formulas written for easier or more efficient computation:

$$P(U = n_A, V = N_D) \times (t + w + 1) = P([X = H_A - n_A d_D > 0] \& [Y = H_D - N_D d_A \leq 0]) \times (t + w + 1)$$

$$= P_D^{n_A} P_A^{N_D} \sum_{i=\max\{1,-s-t\}}^{w-s} \sum_{j=0}^{i} \left(n_A \geq j \ ? \ \binom{i}{j} P_A^{i-j} f(N_D - 1, n_A - j) : 0 \right)$$

$$+ \sum_{i=\max\{0,s-w\}}^{s+t} \sum_{j=0}^{i} \left(N_D > j \ ? \ f(n_A, N_D - j - 1) \binom{i}{j} P_A^{N_D} P_D^{n_A + i - j} : \left[n_A = 0 \ ? \ \binom{i}{j} P_A^j P_D^{i-j} : 0 \right] \right).$$

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$$P(U=N_A, V=n_D)\times(t{+}w{+}1)(1{-}Q) = P([X=H_A{-}N_A d_D \le 0]\&[Y=H_D{-}n_D d_A > 0])\times(t{+}w{+}1)(1{-}Q)$$

$$= \sum_{i=\max\{1,-s-t\}}^{w-s} \sum_{j=0}^{i} \left(N_A > j \ ? \ f(n_D, N_A-j-1) \binom{i}{j} P_D^{N_A} P_A^{n_D+i-j} \ : \ \left[n_D = 0 \ ? \ \binom{i}{j} P_D^{j} P_A^{i-j} \ : \ 0 \right] \right)$$

$$+P_A^{n_D} P_D^{N_A} \sum_{i=\max\{0,s-w\}}^{s+t} \sum_{j=0}^{i} \left(n_D \ge j \ ? \ \binom{i}{j} P_D^{i-j} f(N_A-1, n_D-j) \ : \ 0 \right).$$

$$P(U=N_A{-}1, V=n_D)\times(t{+}w{+}1)Q = P([X=H_A{-}(N_A{-}1)d_D \le 0]\&[Y=H_D{-}n_D d_A > 0])\times(t{+}w{+}1)Q$$

$$= \sum_{i=\max\{1,-s-t\}}^{w-s} \sum_{j=0}^{i} \left(N_A > j \ ? \ f(n_D, N_A-j-1) \binom{i}{j} P_D^{N_A} P_A^{n_D+i-j} \ : \ \left[n_D = 0 \ ? \ \binom{i}{j} P_D^{j} P_A^{i-j} \ : \ 0 \right] \right)$$

$$+P_A^{n_D} P_D^{N_A} \sum_{i=\max\{0,s-w\}}^{s+t} \sum_{j=0}^{i} \left(n_D \ge j \ ? \ \binom{i}{j} P_D^{i-j} f(N_A-1, n_D-j) \ : \ 0 \right).$$

where

$$f(x,y) = \frac{\prod_{k=0}^{\min\{x,y\}-1}(x+y-k)}{\min\{x,y\}!} = \frac{(x+y)!}{x!y!}$$

The point of the function f is to help reduce (greatly) the likelihood of integer overflow for extremely onesided battles, even though one would be using the long integer data type.